Master Course Description for EE-445

Title: Foundations of Optimization and Machine Learning

Credits: 4

Course Catalog Entry:

EE 445: Foundations of Optimization and Machine Learning is an introduction to optimization and advanced machine learning models motivated by their application in areas including statistics, decision-making and control, and communication and signal processing. Topics include convex sets and functions, convex optimization problems and convex modeling, duality, linear and quadratic programming, and basic algorithms such as gradient descent, with emphasis on supervised and unsupervised machine learning problems (e.g., regularized regression, robust classification, clustering, and logistic regression), building upon EE 345 concepts.

Coordinators:

- Maryam Fazel, Professor, Electrical and Computer Engineering
- Lillian Ratliff, Associate Professor, Electrical and Computer Engineering
- Vasileios Charisopoulos, Assistant Professor, Electrical and Computer Engineering

Goals: To give ECE students the foundational mathematical concepts that underpin modern optimization and machine learning algorithms. Provide a background in convex problem modeling and solving. Develop a mathematical understanding of how convex optimization tools are used in the design and analysis of machine learning algorithms and optimization problems used in various ECE application domains including data science, decision-making and control, communication, and signal processing.

Learning Objectives: At the end of this course, students will be able to:

1. Identify and characterize convex sets, functions, and optimization problems.
2. Develop skills to model applied problems as convex problems.
3. Gain experience with the modeling environment CVX/CVXPY to formulate and solve convex problems, implement simple optimization methods such as gradient descent in Python.
4. Explain various machine learning models and algorithms using the language of optimization.

Textbook:

- Main: [Optimization Models in Engineering] (Giuseppe Calafiore and Laurent El Ghaoui)
  - Required
- Supplementary: [Convex Optimization] (Stephen Boyd and Lieven Vandenberghe)
  - Optional – Free Online
- Supplementary: [Introduction to Applied Linear Algebra: Vectors, Matrices, and Least Squares] (Stephen Boyd, Lieven Vandenberghe)
  - Optional – Free Online

Prerequisites by Topic:

1. Calculus sequence: MATH 124, 125, & 126
2. Python: EE 241 or CSE 163
3. EE 345: Introduction to Foundations of Machine Learning

Course Structure Overview:

1. **Module-0: Review of Mathematical Foundations**
   Module-0 consists of a brief review of 345 linear algebra topics. Topics include:
   - Vectors, norms
   - Matrices, eigen decomposition
   - Positive semi-definite matrices, singular value decomposition, and principal component analysis (PCA)
2. **Module-1: Introduction to Convexity**
   Optimization is at the core of every machine learning model. Module-1 concentrates on convex analysis, modeling and optimization with connections to machine learning. In particular, it will be demonstrated that machine learning problems and algorithms arising in different domains can be modeled using the language of convex optimization. Topics include:
   - Convex sets
   - Convex functions
   - Convex optimization problems
3. **Module-2: Duality and Optimality Conditions**
   - Lagrangian duality in convex optimization optimality conditions including Karush-Kuhn-Tucker conditions
   - Linear and quadratic programs

   Applications will be used throughout this module to convey concepts. Examples of applications include:
   - Machine Learning: linear classification, max-margin classification
   - Control and Signal Processing with Applications in Game Theory: finding Nash equilibria in matrix games via linear (zero-sum) and quadratic (general sum) programming, system identification for ARX/ARMA models
   - Quantitative Finance: risk assessment via linear programming

4. Module-3: Convex Optimization Algorithms: Gradient Descent and Variants

Module-3 gives a short overview of gradient-based unconstrained minimization.

- Iterative minimization algorithms overview
- Gradient descent for convex (unconstrained) minimization: step size, convergence; Stochastic gradient descent
- Iterative shrinkage algorithm for l_1 norm penalty
- Examples: Logistic regression, LASSO

5. Module-4: Convex Modeling for Machine Learning and Other Applications

Module-4 combines concepts from the previous three modules by revisiting the example applications in greater detail. The applications will be the primary focus and connections will be drawn to different aspects of the modeling, data analysis and solutions (optimization problem or algorithm) as they relate to the concepts from Modules 1-3. Example applications include:

- Machine Learning: Logistic regression; SVM; Regularized regression–LASSO; Training (simple) Neural Networks
- Control and Signal Processing: sparse signal reconstruction
- Quantitative Finance: mean-variance analysis in portfolio selection and asset pricing; asset allocation

Course Structure: The class meets for two 80-minute lectures and one 2 hour discussion section per week. The latter is administered by teaching assistants. Homework (with theoretical and computational components) is assigned weekly. One exam is given nominally at the end of the 5th week, and a comprehensive final exam is given at the end of the quarter.

Computer Resources: The course uses Python for the computational components of the homeworks, as well as the modeling environments CVX (which can be called within Matlab) or CVXPY (which uses Python). Students are expected to use their personal computers.

Laboratory Resources: None.

Grading: Approximate distribution: Homework 35%, Midterm Exam 25%, Final Exam 40%. The grading scheme in any particular offering is the prerogative of the instructor.

ABET Student Outcome Coverage: This course addresses the following outcomes:

H = high relevance, M = medium relevance, L = low relevance to course.

(1) *An ability to identify, formulate, and solve complex engineering problems by applying principles of engineering, science, and mathematics* **(H)** The homework and exams require direct application of mathematical knowledge to engineering problems, and require students to model engineering problems in the language of convex optimization.

(3) *An ability to communicate effectively with a range of audiences* **(L)** Students will learn and apply techniques to rigorously and formally apply and communicate theoretical concepts.

(7) *An ability to acquire and apply new knowledge as needed, using appropriate learning strategies.*

Prepared By: Maryam Fazel, Lillian Ratliff

Last Revised: 3/3/2025

Textbook table of contents